Technical Brief

Measurements of Mutual Inductance and Capacitance in the Presence of Arbitrary Termination

The derivation of equations for computing mutual inductance and capacitance in a package, connector, or socket were presented in reference [1]. These equations were derived, however, under the condition that the measured traces in the device under test (DUT) are terminated with short terminations for the mutual inductance computation and with open terminations (left open-ended) for mutual capacitance computation.

In many practical cases, however, it is necessary to have a different type of termination at the end of the measured trace, a 50 Ω termination being the simplest example. In this technical brief, we derive the equations for mutual inductance and capacitance measurements under arbitrary termination conditions, and define how C_{mutual} and L_{mutual} computations in IConnect® TDR software are affected by the impedance values of these terminations.

Mutual Inductance

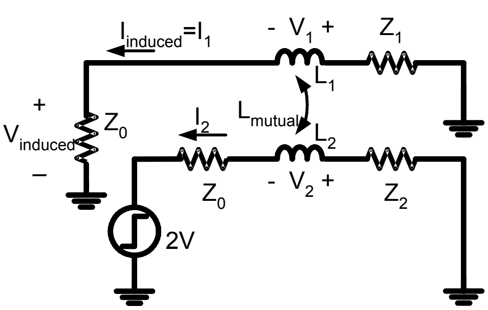

Figure 1. Mutual inductance measurement in presence of arbitrary termination

The coupled inductor system shown above is described by

$$V_1 = L_1 \frac{dI_1}{dt} + L_{mutual} \frac{dI_2}{dt}$$
$$V_2 = L_{mutual} \frac{dI_1}{dt} + L_2 \frac{dI_2}{dt} \quad (1)$$

Integrating the fist equation results in

$$\int_0^\infty V_1 dt = L_{mutual}\left(I_2(\infty) - I_2(0)\right) + L_1\left(I_1(\infty) - I_1(0)\right) \quad (2)$$

Inspecting the above measurement setup, we see that $I_2(0)$, $I_1(\infty)$, and $I_1(0)$ are zero, resulting in

$$L_{mutual} = \frac{\int_0^\infty V_1 dt}{I_2(\infty)} \quad (3)$$

We also see that

$$V_1 = -V_{induced}\left(1 + \frac{Z_1}{Z_0}\right) \quad (4)$$

$$I_2(\infty) = -\frac{2V}{Z_0 + Z_2} \quad (5)$$

Resulting in

$$L_{mutual} = \frac{(Z_0 + Z_1)(Z_0 + Z_2)}{2Z_0 V} \int_0^\infty V_{induced} dt \quad (6)$$

The extracted inductance values are valid so long as Z_1 and Z_2 are finite, which means neither line can be left unterminated. Common termination schemes are $Z_1=Z_2=0$ (short circuited) and $Z_1=Z_2=Z_0$ (matched). Note that if Z_1 or Z_2 become large compared to Z_0, $V_{induced}$ becomes small, resulting in a reduced signal to noise ratio in the measurement system.

In case of $Z_1=Z_2=0$, equation (6) collapses to

$$L_{mutual\ short} = \frac{Z_0}{2V} \int_0^\infty V_{induced} dt \quad (7)$$

which corresponds to the mutual inductance equation derived in [1]. This is the equation that IConnect uses to compute mutual inductance. Therefore, the actual mutual inductance value for the DUT can be recomputed as

$$L_{mutual} = \frac{(Z_0 + Z_1)(Z_0 + Z_2)}{Z_0^2} L_{mutual\ short} \quad (8)$$

Therefore, if the offender line is terminated in 50 Ω ($Z_1 = Z_0 = 50$ Ω), then the inductance value computed in IConnect using equation (7) is half of the actual inductance value L_{mutual}. If both lines are terminated in 50 Ω, then the actual L_{mutual} is 4 times the IConnect computed $L_{mutual\ short}$ value.

Mutual Capacitance

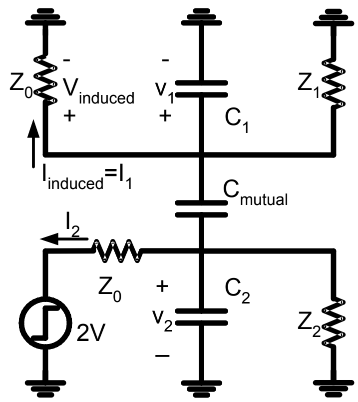

Figure 2. Mutual capacitance measurement in presence of arbitrary termination

The system shown above is described by

$$I_1 = I_{induced} = -(C_1 + C_{mutual})\frac{dV_1}{dt} + C_{mutual}\frac{dV_2}{dt} - \frac{V_1}{Z_1}$$
$$I_2 = C_{mutual}\frac{dV_1}{dt} - (C_2 + C_{mutual})\frac{dV_2}{dt} - \frac{V_2}{Z_2} \qquad (9)$$

Integrating the first equation results in

$$\int_0^\infty I_{induced}\,dt = -(C_1 + C_{mutual})(V_1(\infty) - V_1(0)) + C_{mutual}(V_2(\infty) - V_2(0)) - \int_0^\infty \frac{V_1}{Z_1}dt \qquad (10)$$

Inspecting the above measurement setup, we see that $V_2(0)$, $V_1(\infty)$, and $V_1(0)$ are zero, resulting in

$$C_{mutual} = \frac{\int_0^\infty \left(I_{induced} + \frac{V_1}{Z_1} \right) dt}{V_2(\infty)} \qquad (11)$$

We also see that

$$I_{induced} + \frac{V_1}{Z_1} = \frac{V_{induced}}{Z_0} + \frac{V_{induced}}{Z_1} \qquad (12)$$

$$V_2(\infty) = 2V\left(\frac{Z_2}{Z_0 + Z_2} \right) \qquad (13)$$

Resulting in

$$C_{mutual} = \frac{(Z_0 + Z_1)(Z_0 + Z_2)}{2Z_0 Z_1 Z_2 V}\int_0^\infty V_{induced}\,dt \qquad (14)$$

In case of $Z_1 = Z_2 = \infty$, this equation collapses to

$$C_{mutual\ open} = \frac{1}{2Z_0 V}\int_0^\infty V_{induced}\,dt \qquad (15)$$

which corresponds to the mutual capacitance equation derived in [1]. This is the equation that IConnect uses to compute mutual capacitance. Therefore, the actual mutual capacitance value for the DUT can be recomputed as

$$C_{mutual} = \frac{(Z_0 + Z_1)(Z_0 + Z_2)}{Z_1 Z_2} C_{mutual\ open} \qquad (16)$$

Therefore, if the offender line is terminated in 50 Ω ($Z_1 = Z_0 = 50\ \Omega$), then the inductance value computed in IConnect using equation (16) is half of the actual inductance value C_{mutual}. If both lines are terminated in 50 Ω, then the actual C_{mutual} is 4 times the IConnect computed $C_{mutual\ short}$ value.

Bibliography

[1] *TDR Techniques for Characterization and Modeling of Electronic Packaging*, - High Density Interconnect Magazine, March and April 2001, 2 parts (TDA Systems Application Note PKGM-0101)

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